EE 521 Digital Signal Processors Lab

Assignment 4

4 February 2021

This assignment will be on DFT and DCT. What is the limitation of DTFT? How does DFT eliminates them?

- **Discrete Fourier Transform:** For a signal $x[n]$ with length N, its N-point DFT $X(k)$ is defined as follows:

  $$X(k) = \sum_{k=0}^{N-1} x[n]e^{-j(2\pi/N)kn}$$

  The N-point inverse DFT of $X(k)$ is then given by:

  $$x[n] = \frac{1}{N}\sum_{n=0}^{N-1} X(k)e^{jk(2\pi/N)kn}$$

  Write a function to compute the circular convolution of two sequences. Find the circular convolution of the two sequences $x = [1, 2, 3, 0, 0, 0, 0]$ and $h = [1, 1, 1, 1, 1, 1, 1]$ using the function.

  A signal is given as $x(t) = cos2\pi ft$ where $f = 30$ Hz. The signal is sampled at 100 times per second for 10 seconds and then its DFT is computed.Plot the spectrum versus frequency in Hz. What is the magnitude of DFT at 35 Hz? Is this value non-zero? If so then why?

  Consider the two sequences $x = [1, -3, 1, 5]$ and $y = [7, -7, -9, -3]$. Does there exist a sequence h such that y is the circular convolution of x and h. Find h using DFT and IDFT.

  Compute the energy of the signal $x[n] = (3/4)^n u[n]$ for $0 \le n \le 50$. If the DFT of this signal is $X(k)$ then what is the energy of $X(k)$ and how does it relate to that of $x[n]$.

  Is it possible to compute the DTFS coefficients of the square wave mentioned in Assignment-3 using the function written for DFT as mentioned above. If yes, then modify the function to compute the DTFS coefficients of the signal and show a labelled plot of the spectrum of these coefficients.

- **Discrete Cosine Transform:** It is used in lossy image compression because it has very strong energy compaction, i.e., its large amount of information is stored in very low frequency component of a signal and rest other frequency having very small data which can be stored by using very less number of bits. The general equation for a 1D (N data items) DCT is defined by the following equation:

$$X(m) = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} x[n] c_m cos[\frac{\pi m}{2N}(2n+1)]$$

The inverse transform is,

$$x[n] = \sqrt{\frac{2}{N}} \sum_{m=0}^{N-1} X(m) c_m cos[\frac{\pi m}{2N}(2n+1)]$$

where,

$$c_m = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } m = 0 \\ 1 & \text{otherwise} \end{cases}$$

Consider a signal $x[n] = [8, 16, 24, 32, 40, 48, 56, 64]$ of length $N = 8$. Compute the DCT of this signal. Now keep only the first four DCT coefficients, discarding the rest and then compare its energy with that of $x[n]$. Compute the inverse DCT of the sequence obtained and plot it alongside $x[n]$. Also compute the mean square error between the two signals.

Now repeat the same process with DFT and compare the mean square errors. Comment on your observation.