EE 521 Digital Signal Processors Lab

Assignment 3

27 January 2021

This assignment will be on transforms. Why do we use transforms? Some operations in the time domain are complicated like convolution sum. If we use the transform domain the same complicated operations become easy. We will focus mostly on discrete time signals and their transforms.

- **Z Transform:**

  Consider the discrete time sequence $x[n] = z^n$ with $-\infty < n < \infty$ where $z \in \mathbb{C}$.. Let $z = 0.95$, $z = 1.05$, $z = -1.05$, $z = -0.97$, $z = 1$, $z = -1$. Now plot the sequences for different values of z. For each plot what is the nature of the plot?

  Now consider the same sequence with complex values of z. Consider $z = -2 + 3j$, $z = 2 - 3j$, $z = e^{j\pi/10}$, $z = 0.95e^{j\pi/10}$, $z = 1.05e^{j\pi/10}$. For these values of z create 3D plot with both real and imaginary parts. Try using different markers for real and imaginary parts.

  Consider the system $y[n] = 2x[n-3]$, the input to this system is the sequence z^n where z belong to the sequences in the previous section (all complex). Find the response of the system. Also find the corresponding eigen values of the response.

  The eigen values of the response is the Z Transform of the impulse response. This is also known as the transfer function denoted by $H(z)$. Compute the magnitude spectrum $|H(z)|$ and the phase spectrum $arg[H(z)]$ and plot them in separate plots. Plot for the previous section by considering complex z values from last part.

  Compute the Z Transform for the basic signals: $\delta[n]$, $u[n]$, $r[n]$, $\alpha^n u[n]$, $n\alpha^n u[n]$, $r^n \cos\omega_0 n u[n]$. Choose specific values for each and visualize the magnitude and phase spectrums. Also plot the poles and zeros with the ROC. For which values if any the Z Transform doesnot exist? Try finding out the Z Transform outside the ROC.

Why the pain? See the equations:

$$x[n] = \sum_k a_k z_k^n$$

$$y[n] = \sum_k a_k H(z_k) z_k^n$$

So if we can write every signal as a linear combination of complex exponential by using the eigen value property we can also get the response to the LTI system.

- **Discrete Time Fourier Series:** For a signal $x[n]$ with fundamental period N, the DTFS representation is as follows:

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk(2\pi/N)n}$$

The DTFS coefficients are defined by following relation:

$$x[n] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk(2\pi/N)n}$$

The DTFS coefficients of a periodic discrete time signal with period $N = 5$ are given below:

$$a_0 = 1, a_2 = a_{-2}^* = e^{j\pi/4}, a_4 = a_{-4}^* = 2e^{j\pi/3}$$

Using the properties of DTFS, determine the values of a_0 through a_4.

Using these coefficients, determine one period of the signal and display a labelled plot.

Consider a periodic square wave with fundamental period $N = 16$ having magnitude 1 between $n = 0$ and $n = 7$ and zero elsewhere. Make a plot of this signal over two periods in blue colour.

Determine the DTFS coefficients of this signal and generate magnitude plots of the coefficients.

Using the coefficients determined above, make an approximate reconstruction of the square wave and display it on the plot of the original signal in red colour.

- **Continuous Time Fourier Transform:** Consider a rectangular signal with amplitude 5 and existing between $t = -1$ and $t = 1$. Compute the continuous time Fourier transform of this signal and display a nicely labelled plot.

A message signal $m(t)$ is used to modulate a carrier signal $c(t) = cos 2\pi f_c t$ and the resultant signal is given by $y(t) = m(t)c(t)$. Plot $y(t)$.

Compute the Fourier transform of $m(t)$ and $y(t)$ and display the plots.

$m(t)$ is a triangular signal existing between $t = -1$ and $t = 1$ with a peak amplitude of 1.