EE 521 Digital Signal Processors Lab

Assignment 2

20 January 2021

This assignment is the second prerequisite assignment for DSP Lab. The concept of signals and systems were discussed through many examples. An important signal for this was the **Complex Exponential Signal** e^{st} where $s = \sigma + j\omega$. In this assignment, we will explore the concepts of Probability distributions and randomness in signals.

1. **Probability Distributions** In studying signals, we require the help of random variables to model the randomness in them. This randomness is also useful in predicting the nature of noise.

    - **Continuous distributions** Plot the PDF and CDF for the following distributions by taking at least three different sets of parameters for each. Vary the parameters to get at least 3 different realizations of each: Uniform (a,b), Exponential (λ), Gamma (γ), Beta (β), Gaussian (μ,σ), Standard Normal $(0, 1)$, Chi-squared (k). How many of these distributions are non negative? Calculate the mean median mode and variance and plot it with the PDF in a single plot for one distribution. Also draw the CDFs separately.

      Consider a signal with 2 orthogonal components X, Y. Each component is a random variable that is independent of the other and follows Gaussian distribution with 0 mean and equal variances $\sigma^2 = 4$. Can you find the distribution of the power content of the signal i.e. $Power = X^2 + Y^2$. Which distribution is this? Is it a non negative distribution? Compute the mean median mode and variance and plot with the PDF.

    - **Discrete distributions**

      To plot the discreet distributions use the impulse sequence or impulse train. Write the PMF in terms of linear combination of impulses and then plot. You can write a function to generate a impulse train first. PMF's are of form $P(X = x) = [0, 0.2, 0.1, 0.5, ...]$ therefore PMF can also be expressed as $P(X) = \sum_{k=-\infty}^{\infty} P(x_k)\delta[x - x_k]$

Consider a transmitter that can transmit messages only in the form of 2 bits 0 and 1. The receiver receives a single message sent by the transmitter say X. Since, we do not know before hand which bit was sent X follows a certain distribution. Which distribution is this? Find the PMF and plot it. Also calculate the mean and variance of X. Assume probability for receiving 0(failure) is $p = 0.1, 0.5, 0.8$.

Consider the same transmitter but this time the receiver receives 20 bits. Each bit is independent of the previous one. The probability of receiving a 0 is $p = 0.4$. Let X be the number of 1s (successes) that is received at the receiver's end. What is the distribution followed by X. How is it related to the first distribution? Calculate the mean and variance and plot the PMF.

Consider an experiment in which bits are transmitted sequentially. Each bit is independent of the other. When the first bit arrives the receiver checks for a 1. If it is 1 the receiver takes a decision. If it is a 0 the receiver waits for the next bit. This goes on till the receiver finds out that the bit was 1 and the experiment is dropped once the bit is 0. Let X denote that the bit received was 1 on the k^{th} time (example, X=2 suggests that the bit sequence was $0, 1$ and once 1 was received the experiment was stopped). Find the PMF of X. What is this distribution known as? Calculate the mean and variance and plot the PMF. How is this distribution related to the first distribution.

Consider an experiment in which the receiver receives different bits randomly from many transmitters within a second's interval. Each second it was observed that each second (at any interval) the mean number of bits that were received was 5. Let X be the number of bits received every second (during an interval). What is the distribution that is followed by X. Is there any relation between this and the second distribution that we discussed? Calculate the mean, variance and plot the PMF.

2. **Signal measures**

Plot the following sequences: $x_1[n] = [1, 2, 3, 2, 1, 2, 3, 4, 3, 2, 1]$ $x_2[n] = [1, 2, 1, 2, 1, 2, 1, 2]$ $x_3[n] = [1, 2, 3, 2, 4, 6, 4, 8, 12]$ $x_4[n] = [2, 4, 2, 4, 2, 4, 2, 4]$ $x_5[n] = [1, -2, 3, -2, 1, -2, 3, -4, 3, -2, 1]$ and $x_6[n] = 2 \times x_1[n] + 1$. Calculate the autocorrelation function for a delay of 2 samples. Plot the autocorrelation function and find the power of all the sequences. Also find the cross correlation function for all the pairs for a delay of 2 samples. To compute the autocorrelation function write a python function from scratch with the following signature:

```python
## Signature for autocorrelation.py
def autocorrelation(sequence1, sequence2, delay):
sequence1, sequence2 = np.array()
# Formula for ACF
# Plot the ACF also with the particular delay
return autocorrelation
```

Use any library like scipy, numpy or statsmodel to calculate the autocorrelation function for the same sequences. You will get a total of 15 pairs. Compare the timing of your code and the inbuilt function using timeit for all the pairs and observe which is faster, your approach or the library's inbuilt method.

3. **Noise**

   Generate an uncorrelated uniform random noise sequence, an uncorrelated white noise sequence, and a red noise sequence. Why are they called so? What are their distributions?

   Generate a random sinusoid $x[n] = A\cos(\omega_0 n + \phi)$ where A is uniformly distributed random variable in [0,4] and ϕ is uniformly distributed random variable in $[0, 2\pi]$.

   Generate a noisy sinusoid with amplitude 5 and frequence $\omega = 2\pi$. Use the gaussian noise of mean 0 and variance 1 to be used as an envelope (Envelope is a signal which decides the outer structure whereas sinusoid will be embedded in some way to the gaussian noise).

   Bonus(Ungraded): Try visualizing the effects of noise addition to images and a recorded wavfile. Use the speckle, salt and pepper, gaussian noises in images and just gaussian noise in wavfile. Is there any way to remove this noise? Also visualize any waveforms before and after adding noise to it and how it behaves.